UUCMS. No. | | | | | | | | | | | | |

B.M.S COLLEGE FOR WOMEN
BENGALURU – 560004

III SEMESTER END EXAMINATION – APRIL 2024

M.Sc – MATHEMATICS- NUMERICAL ANALYSIS-II
(CBCS Scheme-F+R)

Course Code MM305T **QP Code: 13005**
Duration: 3 Hours **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. (a) Establish Taylor's series method for $y' = f(x,y),\ x > x_0$ subjected to condition $y(x_0) = y_0$ and obtain the first five terms in the Taylor's series as solution of the equation

   $\frac{dy}{dx} = y + x^3, y(1) = 1$ and find the value at $x = 1.3$.

   (b) Find first three successive approximate solutions of initial value problem $\frac{dy}{dx} = x + y, y(0) = 1$ at $x = 0.5$ by Picard's method. **(7+7)**

2. (a) Find the approximate solution of $y' = x + y, y(0) = 0$ using the modified Euler's method. Find $y(0.2)$ and $y(0.4)$ with $h = 0.2$, and obtain the required first solution from Euler method. (b) Solve the system of differential equations

   $\frac{dy}{dx} = -3y + 2z, y(0) = 0$; $\frac{dy}{dz} = 3y - 4z, z(0) = 0.5$, with $h = 0.2$. Obtain $y(0.4)$ and $z(0.4)$. **(7+7)**

3. Derive the Adam Bashforth and Adam Moulton's third and fourth order method for

   $y' = f(x,y), y(x_0) = y_0$. **(14)**

4. (a) Solve the boundary value problem $y'' + y' + 1 = 0, y(0) = 0, y(1) = 0$ using finite difference method. Take h = 0.25.

   (b)Solve $(x+1)\frac{d^2y}{dx^2} + y = 0, y(0) = 1, y(0.2) = 0$ by shooting method **(7+7)**

5. (a) Solve the Laplace equation $u_{xx} + u_{yy} = 0, 0 \le x, y \le 1$ subject to the condition $u(x,0) = 2x, u(x,1) = 2x - 1, u(0,y) = 0, u(1,y) = 0$. Choose $\triangle x = \triangle y = \frac{1}{3}$ using five-point formula.

(b) Solve the boundary value problem $u_{xx} + u_{yy} = \sin \pi x \sin \pi y,\ 0 \le x, y \le 1$ subjected to the conditions $u = 0$ on the boundary. Take $\triangle x = \triangle y = \frac{1}{3}$.

(7+7)

6. (a) Show that the explicit finite difference method of solving one dimensional heat equation is conditionally stable.

(b) Solve the initial value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, u(x,0) = 1,\ u_x(0,t) = 0 = u_x(1,t),\quad t > 0,$ using Crank Nicolson Scheme. Take $\triangle x = \frac{1}{3}, \triangle t = \frac{1}{27}$. Obtain the solution at second time level.

(7+7)

7. (a) Find the solution of two-dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, 0 \le x, y \le 1$ with conditions $u(x,y,0) = \sin \pi x \sin \pi y,\ \ 0 \le x, y \le 1, u = 0$ on the boundary for $t \ge 0$, using ADI method at first time level, with $\triangle x = \triangle y = \frac{1}{3}, \lambda = \frac{1}{8}$.

(b) Show that the explicit finite difference method of solving one dimensional wave equation is conditionally stable.

(7+7)

8. Solve the hyperbolic partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 \le x \le 1$ with condition $u(x,0) = \sin \pi x\ ,\ u_t(x,0) = 0\ , 0 \le x \le 1$ and $u(0,t) = 0 = u(1,t), t > 0$.

(14)

*****